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Optimal Geostationary Satellite Collocation Using Relative Orbital Element Corrections

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The increasing satellite congestion at the geostationary altitude requires positioning a few satellites in the same geostationary slot, a technique known as *collocation*. In this paper, we develop a geostationary orbit satellite collocation algorithm using relative orbital element corrections, which represent the differences between the impulsive orbital element corrections of any two spacecraft in a geosynchronous slot. The main idea is that formulating the problem of collocation in terms of relative orbital element corrections leaves some of the final values of the orbital elements unconstrained. The freedom rendered by this modeling can be used to find impulsive maneuvers minimizing a given performance index. The minimum distance between satellites that guarantees collision-free motion is incorporated into the design process to find necessary and sufficient conditions for the relative eccentricity and inclination vectors, guaranteeing safe collocation. The proposed collocation algorithm is illustrated in a simulation.

Nomenclature

$\mathbf{A}$	=	system matrix
a	=	semimajor axis, km
D	=	mean longitude drift rate, rad/s
e	=	eccentricity
$\mathbf{e}$	=	eccentricity vector
f	=	true anomaly, rad
G	=	sidereal angle of Greenwich, rad
h	=	orbital angular momentum, km^2/s
I	=	inclination, rad
$\mathbf{I}$	=	inclination vector, rad
M	=	mean anomaly, rad
n	=	mean motion, rad/s
p	=	semilatus rectum, km
R_e	=	equatorial radius, km
r	=	orbit radius, km
$\mathbf{r}$	=	position vector, km
S_i	=	satellite i
s	=	right ascension, rad
$\mathbf{u}$	=	specific force vector, km/s^2
$\mathbf{v}$	=	velocity vector, km/s
α	=	orbital element
$\boldsymbol{\alpha}$	=	orbital elements vector
α^+	=	orbital element after impulsive correction
α^-	=	orbital element before impulsive correction
$\Delta\mathbf{v}$	=	velocity correction vector, km/s
$\Delta(\cdot)$	=	variation of $(\cdot)$
θ	=	declination
λ	=	longitude angle, rad
$\boldsymbol{\lambda}$	=	Lagrange multiplier vector
μ	=	gravitational constant, km^3/s^2
Ω	=	right ascension of the ascending node, rad
ω	=	argument of perigee, rad
$\|\cdot\|$	=	Euclidean vector norm

I. Introduction

THE geostationary orbit (GEO) constitutes a narrow region in space, and is therefore a finite natural resource. Because of the growing number of geostationary satellites and the limited number of geostationary slots, satellite operators locate several vehicles very close to one another in the same geosynchronous slot [1,2], a technique known as *collocation* [3,4]. The subject of satellite collocation has seen increasing research attention in the last decade due to the ever-growing crowding of the GEO region.

The satellite cluster concept (that is, collocation of two or more satellites in a restricted longitude interval on the geostationary orbit) has been under consideration in the past several years for future communication satellite missions. However, locating a few satellites in close proximity requires active stationkeeping to prevent collisions, shadowing, sensor interference, and communication jamming due to look-angle excursions.

Before discussing stationkeeping methods for collocation of a few GEO satellites, we shall briefly mention a few common approaches to single-satellite stationkeeping. One of the stationkeeping methods is longitude and eccentricity control using impulsive corrections [5]. The control is done by a single maneuver. By using this approach, the inclination remains uncontrolled. Romero and Gambi [6] used tangential correction maneuvers to control geostationary satellites within the longitude band. They used optimal control techniques based on the sun-pointing perigee strategy (which only determines the best time of the day for the tangential maneuver). The stationkeeping is formulated as a nonlinear programming problem with inequality constraints, so that the proposed approach provides an optimal way to determine the remaining parameters for the stationkeeping (i.e., the time interval between successive corrections and the value of the eccentricity corrections that globally optimize the computation). A more applied approach to GEO stationkeeping, taking into account the actual propulsion system, was also considered in the literature [7].

An alternative approach employs electric thrusters. The use of electric thrusters transforms the stationkeeping problem into the continuous control realm. Losa et al. [8] presented the problem of electric stationkeeping for geostationary satellites. A linear time-varying model for the dynamics of a geostationary satellite affected by perturbations was derived, and the longitude and latitude stationkeeping problem was then formulated as a constrained linear quadratic optimal control problem.

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The aforementioned stationkeeping methods can be used as a launching pad for the development of collocation strategies. For example, Lee and Choi [9] described a few methods for collocation of two GEO satellites and one inclined geosynchronous (GSO) satellite. The coordinated eccentricity vector and inclination vector separation method was applied for the collocation, and the maneuver schedule was planned to minimize the operational load by avoiding simultaneous maneuvers.

Park et al. [4] proposed an algorithm for collocation of an arbitrary number of satellites by optimally allocating an eccentricity vector projection for each satellite using a virtual dynamical model approach. A change in the number of satellites did not affect the structure of the proposed algorithm.

Some authors examined collocation of GEO and GSO satellites [10] sharing the same mean stationkeeping longitude. The inclination control for the GEO satellite in this case keeps the inclination within a ± 0.05 deg band. The three strategies studied were 1) separation by longitude, 2) separation by eccentricity, and 3) improved monitoring with no separation strategy employed.

For the most part, it has been suggested [11,12] to use the eccentricity vector and inclination vector separation method for collocation; this has also been implemented on operational systems such as Astra [13]. Usually, the maneuver schedule is planned to minimize the operational load by avoiding simultaneous maneuvers [11].

In this work, we propose a new collocation method that relies on the following observation: Formulating the problem of collocation in terms of relative orbital element corrections leaves the final values of the corrected elements unconstrained [14]. This freedom can be used to design optimal impulsive maneuvers for initialization, reconfiguration, and maintenance of collocated satellites. Consequently, we cast the collocation problem as a particular case of *formationkeeping*.

However, because the final semimajor axis of the GEO formation must attain the GEO value, we propose using a *virtual leader* to specify the final value of the semimajor axis an/or other orbital elements as needed. The virtual leader is a reference point in space that is used to steer the entire GEO group into the longitude and latitude deadbands. Our approach, therefore, is a new one: It minimizes a given performance index while satisfying the altitude and relative distance constraints required for collocation using *relative* orbital elements. In this sense, our approach is a generalization of existing optimal GEO satellite collocation methods [6], because it offers an excess freedom for GEO satellite collocation.

For approximating the geostationary orbital dynamics, we study local motions about an ideal GEO. We therefore use *synchronous elements* [3] as the state variables. We find the optimal velocity corrections (in the sense of minimizing the squared l^2-norm of the velocity corrections vector) required to collocate a group of GEO satellites and simulate the resulting relative motion. The minimum distance between satellites, guaranteeing collision-free operation, is taken into account to find necessary conditions for the relative eccentricity and inclination vector geometry. The collocation algorithm is tested in a simulation that includes a J_2 perturbation [15].

II. Background

In this section, we briefly outline the underlying model used for the design of optimal collocation, including the effects of J_2. We also describe the relative orbital element corrections used to develop the optimal collocation method. A more elaborate discussion can be found elsewhere [14].

A. Gauss Variational Equations

The Gauss variational equations (GVEs) model the effect of a control and/or a disturbance acceleration vector, $\mathbf{u} = (u_t, u_n, u_h)^T$, on the osculating orbital elements time derivatives. This vector is represented in the frame $\mathcal{T}$, a rotating tangential-normal frame, centered at the satellite. The fundamental plane is the orbital plane. The unit vector $\hat{\mathbf{t}}$ lies along the spacecraft velocity vector, $\hat{\mathbf{h}}$ coincides with the instantaneous angular momentum vector, and $\hat{\mathbf{n}}$ completes the right-hand setup. Thus, $u_t = \mathbf{u}\cdot\hat{\mathbf{t}}$, $u_n = \mathbf{u}\cdot\hat{\mathbf{n}}$, and $u_h = \mathbf{u}\cdot\hat{\mathbf{h}}$.

For impulsive maneuvers, one way write

$$\dot{\alpha}\Delta t = \Delta\alpha, \qquad u_t\Delta t = \Delta V_t, \qquad u_n\Delta t = \Delta V_n, \qquad u_h\Delta t = \Delta V_h \tag{1}$$

where α is an orbital element; $\Delta\alpha$ is an orbital element correction; ΔV_t, ΔV_n, and ΔV_h are the components of the velocity correction vector in frame $\mathcal{T}$; and the impulsive time interval $\Delta t \to 0$. The impulsive form of the GVEs defines algebraic relationships between the orbital element corrections and the components of the velocity correction vector [16]:

$$\Delta a = \frac{2a^2 v}{\mu}\Delta V_t \tag{2a}$$

$$\Delta e = \frac{1}{v}\left[2(e+\cos f)\Delta V_t - \frac{r}{a}\sin f\,\Delta V_n\right] \tag{2b}$$

$$\Delta I = \frac{r\cos(f+\omega)}{h}\Delta V_h \tag{2c}$$

$$\Delta\Omega = \frac{r\sin(f+\omega)}{h\sin I}\Delta V_h \tag{2d}$$

$$\Delta\omega = \frac{1}{ev}\left[2\sin f\,\Delta V_t + \left(2e + \frac{r}{a}\cos f\right)\Delta V_n\right] - \frac{r\sin(f+\omega)\cos I}{h\sin I}\Delta V_h \tag{2e}$$

$$\Delta M = -\frac{b}{eav}\left[2\left(1+\frac{e^2 r}{p}\right)\sin f\,\Delta V_t + \frac{r}{a}\cos f\,\Delta V_n\right] \tag{2f}$$

where a is the semimajor axis; e is the eccentricity; I is the inclination; Ω is the right ascension of the ascending node; ω is the argument of perigee; M is the mean anomaly; f is the true anomaly; r is the scalar orbit radius; p is the semilatus rectum; the mean motion n satisfies $n = \sqrt{\mu/a^3}$, where μ is the gravitational parameter; and the semiminor axis b satisfies $b = a\sqrt{1-e^2}$.

Consequently, in matrix form, the GVEs (2) for some S_i can be written as

$$\Delta\boldsymbol{\alpha}_i = \mathbf{H}(\boldsymbol{\alpha}_i)\Delta\mathbf{v}_i \tag{3}$$

where

$$\Delta\boldsymbol{\alpha}_i = \begin{bmatrix}\Delta a\\ \Delta e\\ \Delta I\\ \Delta\Omega\\ \Delta\omega\\ \Delta M\end{bmatrix}_i, \qquad \mathbf{H}(\boldsymbol{\alpha}_i) = \begin{bmatrix}\mathbf{c}_a^T\\ \mathbf{c}_e^T\\ \mathbf{c}_I^T\\ \mathbf{c}_\Omega^T\\ \mathbf{c}_\omega^T\\ \mathbf{c}_M^T\end{bmatrix}_i, \qquad \Delta\mathbf{v}_i = \begin{bmatrix}\Delta V_t\\ \Delta V_n\\ \Delta V_h\end{bmatrix}_i \tag{4}$$

where $\mathbf{c}^T$ denotes a coefficient vector of the controlled orbital elements emanating from the GVEs, as seen in Eqs. (2), and $\Delta\mathbf{v}$ is the concomitant impulsive velocity correction vector.

B. Mean Orbital Elements Subject to a J_2 Perturbation

Because of the influence of J_2, the orbital elements may generally exhibit short-periodic, long-periodic, and secular variations. Using first-order averaging, the short-periodic variations can be removed. The long-periodic variations are less significant for long-term stationkeeping. To that end, only the secular variations are important. Under the influence of J_2, the secular constituents of the mean orbital elements are given by [16]

$$\frac{da}{dt} = \frac{de}{dt} = \frac{dI}{dt} = 0 \tag{5a}$$

$$\frac{d\Omega}{dt} = -\frac{3}{2}J_2\left(\frac{R_e}{p}\right)^2 n\cos I \tag{5b}$$

$$\frac{d\omega}{dt} = \frac{3}{4}J_2\left(\frac{R_e}{p}\right)^2 n(5\cos^2 I - 1) \tag{5c}$$

$$\frac{dM}{dt} = n + \frac{3}{4}J_2\left(\frac{R_e}{p}\right)^2 n\sqrt{1-e^2}(3\cos^2 I - 1) \tag{5d}$$

where R_e is the equatorial radius.

C. Relative Orbital Element Corrections

Consider the relative motion of a group of N collocated satellites S_i $(i = 1, \ldots, N)$. The impulsive maneuver is represented by a velocity correction vector performed by each collocated satellite, and *relative orbital element corrections* are used for analyzing the relative motion of the satellites in the presence of impulsive maneuvers [14]. A *relative orbital element correction* of S_i and S_j, is defined by [14]

$$\Delta\alpha_{i,j} = \Delta\alpha_j - \Delta\alpha_i \tag{6}$$

where the orbital element corrections of S_i and S_j in the presence of an impulsive velocity correction are, respectively,

$$\Delta\alpha_i = \alpha_i^+ - \alpha_i^- \tag{7}$$

$$\Delta\alpha_j = \alpha_j^+ - \alpha_j^- \tag{8}$$

where α^- is a value of the orbital element before an impulsive correction, and œ$^+$ is the value of an orbital element after the impulsive correction. Therefore, a relative orbital element correction can be written as

$$\Delta\alpha_{i,j} = \Delta\alpha_j - \Delta\alpha_i = \alpha_j^+ - \alpha_j^- - (\alpha_i^+ - \alpha_i^-) \tag{9}$$

The relative orbital element corrections are indifferent to a particular predefined reference value chosen for an orbital element [14].

If perturbations are present, their effect will generally induce a relative drift. The basic requirement from the collocation control law is to cancel the mean relative drift. When the relative deviation exceeds a maximum allowed value, a velocity correction should be applied. Each velocity correction sets the current value of the orbital element correction to a new value that cancels future drifts. In this case, the corrected orbital elements of S_i and S_j satisfy the following condition:

$$\alpha_j^+ - \alpha_i^+ = \alpha_{i,j}^+ \tag{10}$$

and the relative element correction is

$$\Delta\alpha_{i,j} = \alpha_{i,j}^+ - \alpha_{i,j}^- \tag{11}$$

where $\alpha_{i,j}^+$ is the required *relative orbital element* of S_i and S_j, which is defined as

$$\alpha_{i,j} = \alpha_j - \alpha_i \tag{12}$$

after the impulsive correction ($\alpha_{i,j}^+$ is determined based on the effect of J_2 and collision-free motion, as will be shown in the sequel). Similarly, $\alpha_{i,j}^-$ is the relative orbital element before the impulsive correction.

If there are no perturbations present and the collocation is performed by orbital element matching, then the equations describing the relative orbital elements can be written based on Eq. (3) as [14]

$$\mathbf{A}\boldsymbol{\Delta}\mathbf{v} = \mathbf{b} \tag{13}$$

where

$$\mathbf{A} = \begin{bmatrix} -(\mathbf{c}_\alpha)_1^T & (\mathbf{c}_\alpha)_2^T & \mathbf{0}\cdots & \mathbf{0} \\ \mathbf{0} & -(\mathbf{c}_\alpha)_2^T & (\mathbf{c}_\alpha)_3^T\cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \mathbf{0}\cdots & -(\mathbf{c}_\alpha)_{N-1}^T & (\mathbf{c}_\alpha)_N^T \end{bmatrix} \tag{14}$$

is an $(N-1) \times 3N$ matrix, and

$$\mathbf{b} = \begin{bmatrix} \Delta\alpha_{1,2} \\ \Delta\alpha_{2,3} \\ \vdots \\ \Delta\alpha_{N-1,N} \end{bmatrix} \tag{15}$$

is a $(N-1) \times 1$ vector.

It can be seen that the use of relative orbital element corrections introduced an excess freedom [the number of unknowns in Eq. (13) exceeds the number of constraints]. This implies that a static parameter optimization problem with equality constrains can be solved [14].

One option is to use this excess freedom to optimally balance the fuel consumption between the collocated satellites. Ideally, this would required minimizing an L^∞ (min-max) performance index of the form $\mathcal{J} = \|\Delta\mathbf{v}_i\|_\infty$ subject to $\mathbf{A}\boldsymbol{\Delta}\mathbf{v} = \mathbf{b}$. However, this problem cannot be solved analytically. Instead, one may solve an approximate fuel-balanced optimal collocation problem, which can be cast as follows: Find an optimal impulsive maneuver $\Delta\mathbf{v}^*$ satisfying

$$\Delta\mathbf{v}^* = \arg\min_{\Delta\mathbf{v}}\|\Delta\mathbf{v}\|_2^2 = \arg\min_{\Delta\mathbf{v}}\Sigma_i\|\Delta\mathbf{v}_i\|_2^2 \quad \text{subject to } \mathbf{A}\boldsymbol{\Delta}\mathbf{v} = \mathbf{b} \tag{16}$$

This optimization index penalizes large individual fuel consumption because it is proportional to the total mass of propellant squared.

It can be shown that the solution to the optimization problem formulated in Eq. (16) is [14]

$$\Delta\mathbf{v}^* = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\mathbf{b} = \mathbf{A}^+\mathbf{b} \tag{17}$$

where $\mathbf{A}^+$ is the pseudoinverse of $\mathbf{A}$.

III. Geostationary Orbit

A perfect GEO is a mathematical abstraction that could be achieved only by a satellite orbiting a perfectly symmetric Earth with no other forces acting on the satellite other than gravity. In an ideal GEO, a satellite orbits the Earth in the equatorial plane so that the orbital period is equal to the period of the rotation of the Earth.

The Earth completes one rotation in a sidereal day, which amounts to 23 h, 56 min, and 4 s; therefore, its angular velocity is given by

$$\psi = 360.9856747 \text{ deg/day} = 0.729211585 \times 10^{-4} \text{ rad/s} \tag{18}$$

and the resulting radius of the GEO is

$$r = \sqrt[3]{\frac{\mu}{\psi^2}} = 42,164.5 \text{ km} \tag{19}$$

The orbital elements of the ideal GEO are as follows:

$$a_{\text{GEO}} = 42,164.5 \text{ km}, \qquad e = 0, \qquad I = 0 \tag{20}$$

and the angles Ω and ω are undefined. The instantaneous position of the satellite, $\mathbf{r}$, can be represented in Earth-centered inertial coordinates $\mathcal{I}$ as

$$\mathbf{r} = \begin{bmatrix} X_{\mathcal{I}} \\ Y_{\mathcal{I}} \\ Z_{\mathcal{I}} \end{bmatrix} = \begin{bmatrix} r\cos\theta\cos s \\ r\cos\theta\sin s \\ r\sin\theta \end{bmatrix} \tag{21}$$

where s is the right ascension (sidereal angle), θ is the declination, and $r = |\mathbf{r}|$. The location of a GEO satellite is usually specified by the *longitude angle* λ, because it is the only free parameter for different GEO missions. This angle is given by (see Fig. 1)

$$\lambda = s - G \tag{22}$$

where G is the sidereal angle of Greenwich.

Even if the satellite is injected into an ideal GEO, natural perturbations such as gravitational forces due to the sun and the moon and Earth's oblateness affect the initial orbital element values of the orbit. In practice, a GEO is allowed to slightly deviate from the nominal orbit, depending on the type of mission.

A. Synchronous Orbital Elements

The satellite's motion about the ideal GEO can be approximated by using a series expansion [17]. The approximated motion is represented as a deviation of the orbital elements relative to a nominal reference orbit.

To define the position in space of a GEO satellite, let a_{GEO} denote the ideal GEO semimajor axis. The deviation of the semimajor axis of an actual orbit from the ideal GEO value is represented by

$$\delta a = a - a_{\text{GEO}} \tag{23}$$

Next, we define a three-dimensional inclination vector:

$$\mathbf{I} = [\sin I \sin\Omega, -\sin I\cos\Omega, \cos I]^T \tag{24}$$

A three-dimensional inclination vector is a unit vector orthogonal to the orbital plane, for which the direction is aligned with the direction of the orbital angular momentum. It is positive with respect to the motion of the satellite along its orbit. The two-dimensional inclination vector is a projection of the three-dimensional vector on the equatorial plane of the Earth. For small inclinations, $\sin i \approx i$, the two-dimensional inclination vector becomes

$$\mathbf{I} \approx [I\sin\Omega, -I\cos\Omega]^T = [I_x, I_y]^T \tag{25}$$

This vector has magnitude I and it points in the direction $\Omega - 90$ deg. This approximation is valid for GEOs because these are equatorial orbits having very small inclinations. We will therefore use the two-dimensional inclination vector.

We can also define a two-dimensional eccentricity vector,

$$\mathbf{e} = [e\cos(\Omega+\omega), e\sin(\Omega+\omega)]^T = [e_x, e_y]^T \tag{26}$$

which can be visualized as vector on the equatorial plane of the Earth with magnitude e pointing from the center of the Earth to the direction of the perigee. Finally, we define a new set of orbital elements replacing the classical orbital elements:

$$\boldsymbol{\alpha} = (\lambda_0, D, e_x, e_y, I_x, I_y)^T = (\lambda_0, D, \mathbf{e}, \mathbf{I})^T \tag{27}$$

where λ_0 is the mean longitude at the epoch t_0:

$$\lambda_0 = \Omega + \omega - G_0 + (1+D)\psi(t_0 - t_p) \tag{28}$$

where t_p is time of perigee passage, G_0 is the sidereal angle of the Greenwich meridian at epoch, and D is the mean longitude drift rate, given by [3]

$$D = -1.5\frac{\delta a}{a_{\text{GEO}}} \tag{29}$$

and is a measure of the deviation between the orbital period and the rotation of the Earth. The set of orbital elements described in Eq. (27) is called *synchronous elements*. The synchronous elements are defined as osculating elements for perturbed orbits in a manner analogous to that employed for the classical elements.

B. Collocation Methods

Stationkeeping of GEO satellites is performed to confine the subsatellite longitude and latitude into a rectangular box, for which the sides are the longitude and latitude *deadbands* [3], as shown in Fig. 2.

The purpose of collocation is to keep a number of satellites within a single predefined latitude–longitude deadband. Among the different ways to deal with collocation, one can distinguish between four main approaches [3]. In the first approach, each satellite is collocated independently while ignoring the collision risk, so that collocation may be performed only for short periods of times. Another approach is to operate each satellite independently and to correct the relative orbit only if a collision is predicted (this is done by propagating each orbit forward in time). Alternatively, one may separate the collocated satellite by assigning a predefined domain for each satellite, inside which its stationkeeping is performed. This can be implemented by separating the inclination and eccentricity vectors. Finally, collocation can be performed in the relative sense by coordinating the stationkeeping of one or more collocated satellites relative to a *leader* (either real or virtual).

In the next section, we shall discuss an optimal implementation of collocation by combining the separation and coordinated stationkeeping methods.

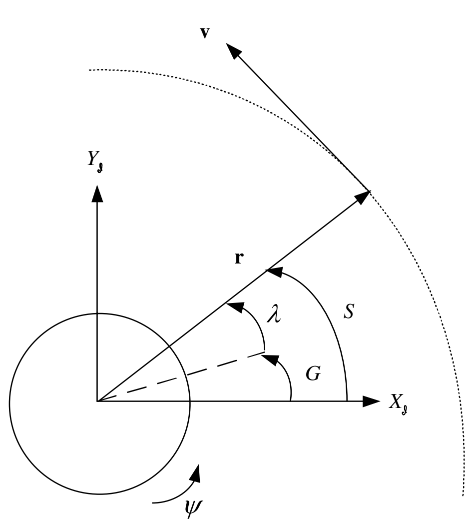

Fig. 1 Sidereal angles of Greenwich G, right ascension s, and satellite longitude λ as seen from the north.

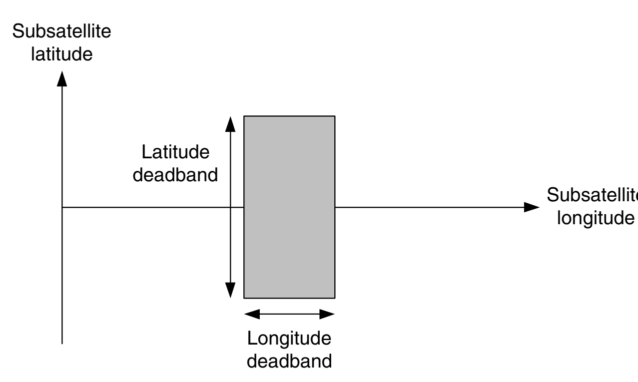

Fig. 2 GEO satellite deadband parameters.

IV. Optimal Collocation by Separation

To implement our proposed optimal collocation method, we will use relative synchronous orbital element corrections (cf. Sec. II.C). An important criterion to be checked is the distance between collocated satellites. This distance can be calculated in the following manner.

The synchronous orbital elements for a satellite S_i are given by

$$\boldsymbol{\alpha}_i = [\lambda_{0_i}, D_i, \mathbf{e}_i, \mathbf{I}_i]^T \tag{30}$$

and so the relative synchronous orbital elements of S_i and S_j can be written as [cf. Eq. (12)]

$$\boldsymbol{\alpha}_{i,j} = \boldsymbol{\alpha}_j - \boldsymbol{\alpha}_i = [\lambda_{0_{i,j}}, D_{i,j}, \mathbf{e}_{i,j}, \mathbf{I}_{i,j}]^T \tag{31}$$

Based on the relative orbital elements, one can write the differences between the satellite positions in the radial, along–track, and cross–track directions, respectively [3]:

$$r_{i,j} = -a_{\text{GEO}}[\tfrac{2}{3}D_{i,j} + e_{x_{i,j}} \cos s + e_{y_{i,j}} \sin s] \tag{32a}$$

$$a_{\text{GEO}}\lambda_{i,j} = a_{\text{GEO}}\lambda_{0_{i,j}} + a_{\text{GEO}}D_{i,j}(s - s_0) + 2a_{\text{GEO}}[e_{x_{i,j}} \sin s - e_{y_{i,j}} \cos s] \tag{32b}$$

$$a_{\text{GEO}}\theta_{i,j} = -a_{\text{GEO}}[I_{x_{i,j}} \cos s + I_{y_{i,j}} \sin s] \tag{32c}$$

Note that in Eqs. (32), the relative right ascension is defined as

$$s_{i,j} = \tfrac{1}{2}(s_i + s_j) \tag{33}$$

Finally, the distance between two collocated satellites is

$$d_{i,j} = \sqrt{r_{i,j}^2 + (a_{\text{GEO}}\lambda_{i,j})^2 + (a_{\text{GEO}}\theta_{i,j})^2} \tag{34}$$

A. Combined Inclination and Eccentricity Separation in the Meridian Plane and Collision Avoidance

The out-of-plane separation can be achieved by setting the relative inclination vector between any two satellites, S_i and S_j:

$$\mathbf{I}_{i,j}^+ = \mathbf{I}_j^+ - \mathbf{I}_i^+ \neq \mathbf{0} \tag{35}$$

This condition is not sufficient to ensure separation, because a collision can occur twice per sidereal day when both satellites pass through the point of crossing of their orbital planes. Thus, additional separation measures must be taken at the orbital plane crossings; the out-of-plane separation must be accompanied by an in-plane separation using the relative eccentricity vector:

$$\mathbf{e}_{i,j}^+ = \mathbf{e}_j^+ - \mathbf{e}_i^+ \neq \mathbf{0} \tag{36}$$

By setting $D_{i,j}^+ = 0$, we obtain the relative motion of two satellites in the $(r, a_{\text{GEO}}\theta)$ plane. The distance between their projections onto this plane is

$$r_{i,j}^+ = -a_{\text{GEO}}(e_{x_{i,j}}^+ \cos s + e_{y_{i,j}}^+ \sin s) \tag{37a}$$

$$a_{\text{GEO}}\theta_{i,j}^+ = -a_{\text{GEO}}(I_{x_{i,j}}^+ \cos s + I_{y_{i,j}}^+ \sin s) \tag{37b}$$

$$d_{i,j}^+ = \sqrt{(r_{i,j})^{+^2} + (a_{\text{GEO}}\theta_{i,j})^{+^2}} \tag{37c}$$

The minimum distance between satellites can be calculated by the methods of the theory of quadratic forms [3]:

$$d_{\min} = \min_s d_{i,j} = a_{\text{GEO}}\sqrt{B_1 - \sqrt{B_1^2 + B_2^2}} \tag{38}$$

where

$$B_1 = 0.5(|\mathbf{I}_{i,j}^+|^2 + |\mathbf{e}_{i,j}^+|^2), \qquad B_2 = e_{x_{i,j}}^+ I_{y_{i,j}}^+ - e_{y_{i,j}}^+ I_{x_{i,j}}^+ \tag{39}$$

The necessary and sufficient condition for $d_{\min} \neq 0$, ensuring collision-free separation, is

$$e_{x_{i,j}}^+ i_{y_{i,j}}^+ - e_{y_{i,j}}^+ i_{x_{i,j}}^+ \neq 0 \tag{40}$$

This criterion implies that $\mathbf{I}_{i,j}^+$ and $\mathbf{e}_{i,j}^+$ must not be equal to zero and must not be parallel or antiparallel.

To implement the preceding collocation condition, we can control the semimajor axis, eccentricity, and inclination. This leads to a relationship in the form of Eq. (13). Let us define the matrices $\mathbf{A}_a$, $\mathbf{A}_e$, and $\mathbf{A}_I$ representing the mapping of the velocity corrections to the relative classical orbital element differences $a_{i,j}$, $e_{i,j}$, and $I_{i,j}$ $(i = 1, \ldots, N)$ based on the GVEs (2):

$$\mathbf{A}_a = \begin{bmatrix} -(c_a)_1 & (c_a)_2 & 0\cdots & \mathbf{0}_{(1\times N)} & \mathbf{0}_{(1\times N)} \\ 0 & -(c_a)_2 & (c_a)_3\cdots & \mathbf{0}_{(1\times N)} & \mathbf{0}_{(1\times N)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0\cdots & -(c_a)_{N-1} & (c_a)_N & \mathbf{0}_{(1\times N)} & \mathbf{0}_{(1\times N)} \end{bmatrix} \tag{41}$$

where $(c_a)_i$ is given by [cf. Eqs. (2)]

$$(c_a)_i = \frac{2v_i a_i^2}{\mu} \tag{42}$$

Similarly,

$$\mathbf{A}_e = \begin{bmatrix} -(c_e)_1 & (c_e)_2 & 0\cdots & -(s_e)_1 & (s_e)_2 & 0\cdots & \mathbf{0}_{(1\times N)} \\ 0 & -(c_e)_2 & (c_e)_3\cdots & 0 & -(s_e)_1 & (s_e)_3\cdots & \mathbf{0}_{(1\times N)} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \end{bmatrix} \tag{43}$$

where

$$(c_e)_i = \frac{2(e_i + \cos f_i)}{v_i}, \qquad (s_e)_i = -\frac{r_i \sin f_i}{v_i a_i} \tag{44}$$

and

$$\mathbf{A}_I = \begin{bmatrix} \mathbf{0}_{(1\times N)} & \mathbf{0}_{(1\times N)} & -(k_I)_1 & (k_I)_2 & 0\cdots \\ \mathbf{0}_{(1\times N)} & \mathbf{0}_{(1\times N)} & 0 & -(k_I)_2 & (k_I)_3\cdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \end{bmatrix} \tag{45}$$

where

$$(k_I)_i = \frac{r_i \cos(f_i + \omega_i)}{h_i} \tag{46}$$

We can now implement the chosen collocation method using the algorithm of least squares as given in Eq. (17). The resulting collocation maneuver is the minimum velocity correction to be applied to satisfy the requirement given in Eq. (40).

Finally, an optimal velocity correction vector required for collocation is

$$\begin{bmatrix} (\Delta V_t)_1 \\ (\Delta V_t)_2 \\ \vdots \\ (\Delta V_t)_N \\ (\Delta V_n)_1 \\ (\Delta V_n)_2 \\ \vdots \\ (\Delta V_n)_N \\ (\Delta V_h)_1 \\ (\Delta V_h)_2 \\ \vdots \\ (\Delta V_h)_N \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a \\ \mathbf{A}_e \\ \mathbf{A}_I \end{bmatrix}^+ \begin{bmatrix} a^-_{1,2} + 0 \\ a^-_{2,3} + 0 \\ \vdots \\ a^-_{N,N-1} + 0 \\ e^-_{1,2} + e^+_{1,2} \\ e^-_{2,3} + e^+_{2,3} \\ \vdots \\ e^-_{N,N-1} + e^+_{N,N-1} \\ I^-_{1,2} + I^+_{1,2} \\ I^-_{2,3} + I^+_{2,3} \\ \vdots \\ I^-_{N,N-1} + I^+_{N,N-1} \end{bmatrix} \tag{47}$$

or, written using vector representation,

$$\Delta \mathbf{v}^* = \mathbf{A}^+(\boldsymbol{\alpha}^-_{i,j} + \boldsymbol{\alpha}^+_{i,j}) = \mathbf{A}^+\mathbf{b} \tag{48}$$

where $\mathbf{A}_a$, $\mathbf{A}_e$, and $\mathbf{A}_I$ are $3N \times (N-1)$ matrices given by Eqs. (41–45), and $\mathbf{b}$ is a superposition of the precorrected relative orbital elements vector $\boldsymbol{\alpha}^-_{i,j}$ and the final relative elements vector $\boldsymbol{\alpha}^+_{i,j}$.

The velocity correction obtained by Eq. (47) is a function of true anomaly f. It was previously shown [14] that the semimajor axis and eccentricity should be corrected at $f = 270$ deg and the inclination correction should be carried out at $f + \omega = 180$ deg. The collocation maneuver thus comprises inclination correction at $f + \omega = 180$ deg and then a semimajor axis and eccentricity correction at $f = 270$ deg. In addition, we note that the collocation maneuver (48) may be performed whenever a violation of the collocation constraints is detected; indeed, in real applications, the proposed collocation maneuver would be performed every few weeks to account for third-body and solar radiation pressure perturbations. The issue of periodic formationkeeping maneuvers is thoroughly discussed elsewhere [14].

V. Numerical Examples

A. Perturbation-Free Case

The following example illustrates an optimal collision-free collocation of four GEO satellites using a combined inclination and eccentricity separation in the meridian plane. Four satellites are located on an almost ideal GEO. We wish to generate an optimal impulsive maneuver for this group of satellites to collocate them in the same slot. Initial orbital element values for the group are summarized in Table 1.

An important parameter in the collocation problem is the relative distance between collocated satellites. This distance oscillates during the orbital cycle according to Eq. (32). A close distance can cause a collision between satellites and a very large distance can cause communication problems. We assume that the minimum distance should be larger than 5 km and that the maximum allowed distance is 100 km. The relative distance can be changed by modifying the final relative eccentricity and inclination vectors. The final relative eccentricity and inclination vectors in the current example are as follows:

$$e^+_{1,2} = 0.0001, \qquad e^+_{2,3} = 0.00012, \qquad e^+_{3,4} = 0.000011$$
$$I^+_{1,2} = 0.0001 \text{ rad}, \qquad I^+_{2,3} = 0.0001 \text{ rad}, \qquad I^+_{3,4} = 0.0001 \text{ rad} \tag{49}$$

Table 1 Initial conditions for collocation

Spacecraft	a, km	e	i, rad	Ω, rad	ω, rad	f, rad	λ_0, rad
S_1	42,164.3	0.0011	0.01	4.0104	0	4.712	2.2562
S_2	42,164.4	0.0011	0.01	4.0062	0	4.712	2.252
S_3	42,164.5	0.0011	0.01	4.0020	0	4.712	2.2478
S_4	42,168.6	0.0011	0.01	3.9993	0	4.712	2.2451

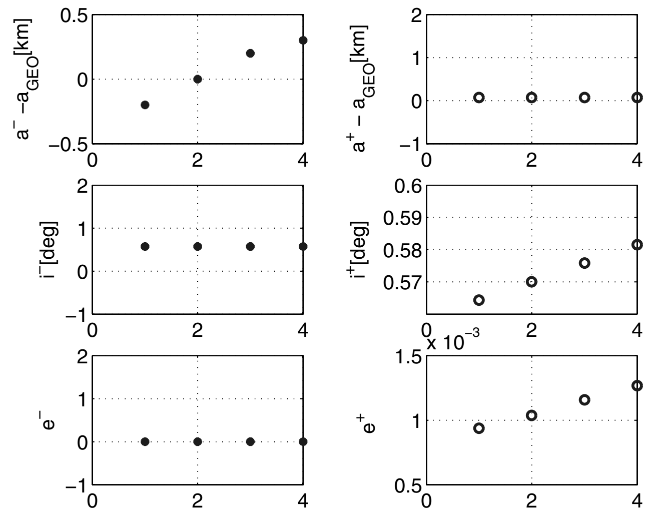

Fig. 3 Collocation by in-plane separation: semimajor axis and mean longitude matching. The figure shows the semimajor axis (relative to the ideal GEO value), inclination, and eccentricity before (left) and after (right) the collocation maneuver.

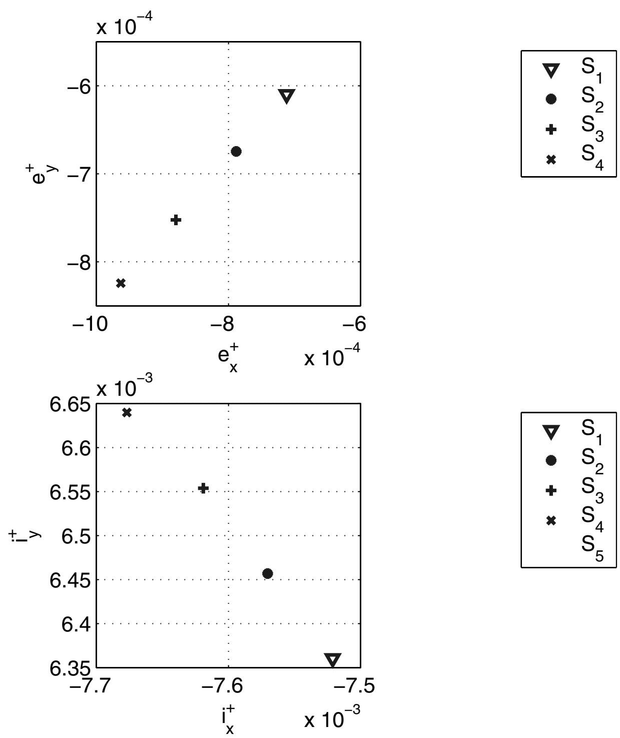

Fig. 4 Collocation by in-plane separation: two-dimensional eccentricity (top) and inclination (bottom) vectors. Eccentricity and inclination separation guarantees collision-free operation.

The results of this collocation maneuver are shown in Figs. 3 and 4. Figure 3 shows the semimajor axis, eccentricity, and inclination for each of the four satellites before and after the collocation maneuver. This collocation maneuver matches the semimajor axis and sets the final relative eccentricity and inclination vectors to the values that will prevent collisions and will satisfy the condition given by Eq. (40).

Figure 4 shows the planes of the two-dimensional eccentricity [Eq. (26)] and inclination [Eq. (25)] vectors. The different values of e^+_x and e^+_y of each satellite cause in-plane separation, and I^+_x and I^+_y are responsible for the out-of-plane separation.

The minimum and maximum relative distances, depicted by Fig. 5, correspond to the prespecified values. These relative distances

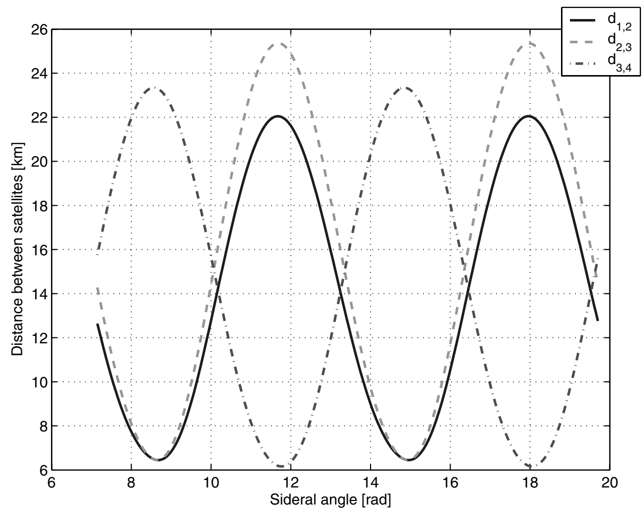

Fig. 5 Relative distance during an orbital period after the collocation maneuver. The minimum and maximum values lie within the prespecified domain.

can be controlled by changing the final eccentricity and inclination vector differences.

The total velocity correction of each satellite, calculated as

$$\Delta V_{\mathrm{tot}_i} = \sqrt{\Delta V_{t_i}^2 + \Delta V_{h_i}^2 + \Delta V_{n_i}^2}$$

and the overall velocity correction required for the collocation maneuver are as follows:

$$\Delta V_{\mathrm{tot}_1} = 0.96 \text{ m/s}, \qquad \Delta V_{\mathrm{tot}_2} = 0.3457 \text{ m/s} \tag{50a}$$

$$\Delta V_{\mathrm{tot}_3} = 0.3305 \text{ m/s}, \qquad \Delta V_{\mathrm{tot}_4} = 1.8607 \text{ m/s} \tag{50b}$$

$$\Delta V_{\mathrm{tot}} = \Delta V_{\mathrm{tot}_1} + \Delta V_{\mathrm{tot}_2} + \Delta V_{\mathrm{tot}_3} + \Delta V_{\mathrm{tot}_4} = 3.4969 \text{ m/s} \tag{50c}$$

B. Virtual Leader

In the preceding example, we illustrated the control of relative orbital elements using relative orbital element corrections. Because the final values of the orbital elements were not specified, the linear least-squares solution yielded an optimal solution in the sense of minimizing the l^2-norm. However, for GEO satellites it is required that the semimajor axis be equal to the ideal value a_{GEO} (deviation of the semimajor axis from the ideal value causes a drift in mean longitude).

Therefore, we will develop a method for forcing the collocated satellites into prespecified semimajor axis values. This will be done by using a *virtual leader*. A virtual leader is a virtual point in space, relative to which one can locate a group of satellites by choosing the appropriate orbital elements of a given satellite relative to the virtual leader. By using this method, the freedom of setting the final value is lost, and so this additional constraint costs an additional amount of fuel.

We will illustrate this method using the group of satellites for which the initial conditions are shown in Table 1. In the current example, however, we will specify the final values of the corrected elements according to the following rationale. The final semimajor axis should be equal to the ideal GEO value $a_{\mathrm{GEO}} = 42,164.5$ km to prevent longitude drift, the final value of the eccentricity should be smaller than 0.001, and the final value of the inclination should be smaller than 0.01 rad. To satisfy these conditions, the desired relative orbital elements are

$$a_{1,2}^+ = -18 \text{ km}, \qquad a_{2,3}^+ = a_{3,4}^+ = a_{4,5}^+ = 0 \tag{51a}$$

$$e_{1,2}^+ = -0.0055, \quad e_{2,3}^+ = 0.0001, \quad e_{3,4}^+ = 0.00015, \quad e_{4,5}^+ = -0.00035 \tag{51b}$$

$$\begin{aligned} I_{1,2}^+ &= -0.0012 \text{ rad}, & I_{2,3}^+ &= -0.0015 \text{ rad} \\ I_{3,4}^+ &= -0.001 \text{ rad}, & I_{4,5}^+ &= -0.0011 \text{ rad} \end{aligned} \tag{51c}$$

where the orbital elements of the virtual leader (numbered as S_1) are as follows:

$$\begin{aligned} a_1 &= 42,181 \text{ km}, & e_1 &= 0.00181, & I_1 &= 0.01 \text{ rad} \\ \omega_1 &= 0 \text{ rad}, & \Omega_1 &= 4.0108 \text{ rad}, & f_1 &= 4.712 \text{ rad} \end{aligned} \tag{52}$$

The initial values and the corrected orbital elements are shown in Fig. 6. Because S_1 is a virtual leader, the particular values of its orbital elements are not important. The important observation is how all the other satellites are located relatively to the virtual leader. As can be seen in Fig. 6, the final semimajor axis value is equal to $a_{\mathrm{GEO}} = 42,164.5$ km.

The required velocity change for this collocation is larger than in the previous example (because we force all three orbital elements to some required values):

$$\Delta V_{\mathrm{tot}} = 45.5 \text{ m/s} \tag{53}$$

Figure 7 depicts the corrected eccentricity and inclination vector components. The circle radius is the maximum-allowed eccentricity and inclination; the corrected values remain inside this circle.

The relative distances after the collocation maneuver between the collocated satellites is shown in Fig. 8. It is evident that a minimum safety distance as well as a maximum distance are maintained throughout the satellite operational time.

Note that one can use the virtual leader approach to constrain the final value of the semimajor axis only. This will relieve some of the excess fuel consumption while leaving some of the freedom rendered by the relative orbital elements differences approach for collocation.

C. Collocation Subject to a J_2 Perturbation

In this section, we will analyze the effect of J_2 on the relative synchronous elements of collocated satellites based on Eqs. (5). We emphasize that this particular perturbation was chosen for illustration purposes only. The collocation algorithm can naturally accommodate any orbital perturbation. The only difference will be the actual values of **b** used for calculating the collocation maneuver.

The Earth oblateness perturbation affects only the nodal rate $\dot{\Omega}$ and the mean latitude rate $\dot{M} + \dot{\omega}$ (assuming first-order averaging and secular effects only). One of the methods that can be used for canceling the relative deviations of the mean drift of $\dot{\Omega}$ and $\dot{M} + \dot{\omega}$ is to match the semimajor axis, eccentricity, and inclination [14]. On the other hand, the preceding collocation conditions were chosen to prevent a collision between two collocated satellites by eccentricity and inclination vector separation, meaning that $e_{i,j}^+ \neq 0$ and $I_{i,j}^+ \neq 0$. One can therefore ask how to combine the collocation conditions with J_2-invariant orbits. The answer is that to fulfill the J_2-invariant orbit condition, one can match the semimajor axis, eccentricity, and inclination [18].

To satisfy the collocation condition, we can use the remaining free parameter: the longitude deadband. If satellites are separated inside the deadband, then the expressions for the relative mean longitude (assuming that $t_0 = t_p$ and $a_i^+ = a_j^+ \to D_{i,j}^+ = 0$) assume the following form:

$$\begin{aligned} \lambda_{0i,j}^+ &= \lambda_{0j}^+ - \lambda_{0i}^+ \\ &= (\Omega_j^+ + \omega_j^+ - G_0) - (\Omega_i^+ + \omega_i^+ - G_0) \\ &= \Omega_{i,j}^+ + \omega_{i,j}^+ \end{aligned} \tag{54}$$

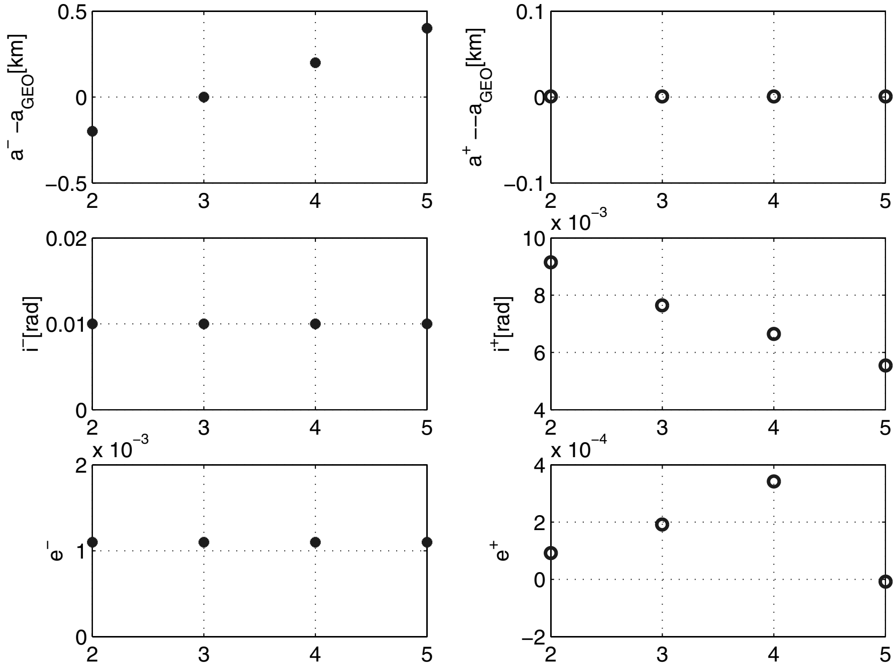

Fig. 6 Collocation by in-plane separation, forcing final conditions. Using the virtual leader technique, all satellites can be collocated at the GEO semimajor axis and desired eccentricity and inclinational separation. The orbital elements of S_1 are not shown because it is a virtual leader.

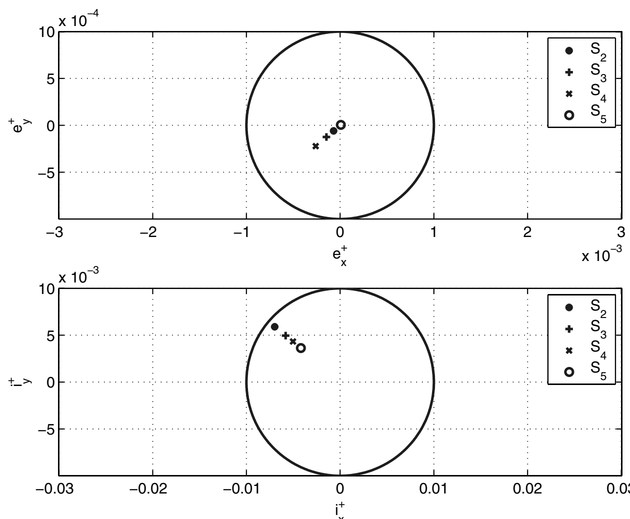

Fig. 7 Collocation by in-plane separation, forcing eccentricity and inclination vectors. The orbital elements of the satellites remain within the prespecified circular projections.

Thus, the relative two-dimensional inclination and eccentricity vectors become, respectively,

$$
\begin{aligned}
\mathbf{I}^+_{i,j} &= [I^+ \sin\Omega^+_j, -I^+\cos\Omega^+_j]^T - [I^+\sin\Omega^+_i, -I^+\cos\Omega^+_i]^T \\
&= [I^+(\sin\Omega^+_j - \sin\Omega^+_i), -I^+(\cos\Omega^+_j - \cos\Omega^+_i)]^T
\end{aligned} \tag{55}
$$

and

$$
\begin{aligned}
\mathbf{e}^+_{i,j} &= [e^+\cos(\Omega^+_j + \omega^+_j), e^+\sin(\Omega^+_j + \omega^+_j)]^T \\
&\quad - [e^+\cos(\Omega^+_i + \omega^+_i), e^+\sin(\Omega^+_j + \omega^+_i)]^T \\
&= \{e^+[\cos(\Omega^+_j + \omega^+_j) - \cos(\Omega^+_i + \omega^+_i)], e^+[\sin(\Omega^+_j + \omega^+_j) \\
&\quad - \sin(\Omega^+_i + \omega^+_i)]\}^T
\end{aligned} \tag{56}
$$

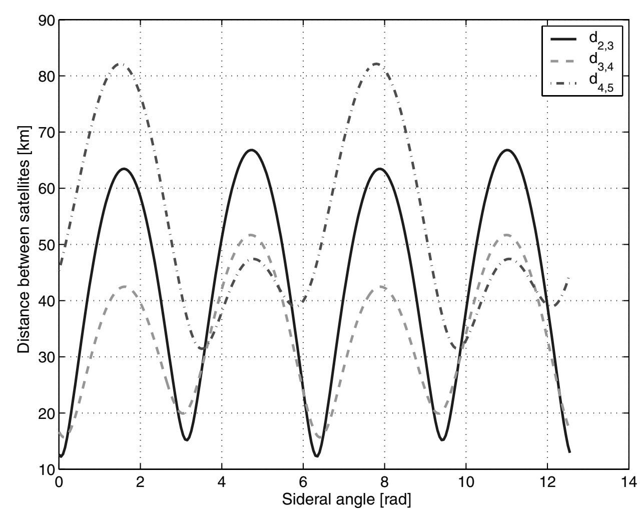

Fig. 8 Relative satellite distances during an orbital period after the collocation maneuver. Minimum safety distances and maximum distances are not violated.

The necessary condition to fulfill the collocation condition is that $\Omega^+_i \neq \Omega^+_j$. This condition will lead to longitude separation and to eccentricity and inclination vector separation, yielding $\mathbf{I}^+_{i,j} \neq 0$ and $\mathbf{e}^+_{i,j} \neq 0$. In other words, we managed to derive a J_2-invariance principle subject to inclination and eccentricity vector separation based on relative orbital elements only.

To validate this statement, we will depict the relative distance of the collocated group of satellites given in Sec. V.A after the collocation maneuver. Figure 9 depicts the relative distance between the satellites for a period of 1 year. It is seen that the minimum and maximum distances remain constant and that the relative orbit is J_2-invariant. The minimum distance can be changed by changing the values of the relative mean longitude λ_0.

VI. Conclusions

An algorithm for collocation of N geostationary orbit (GEO) satellites in the same geostationary slot was developed. Synchronous

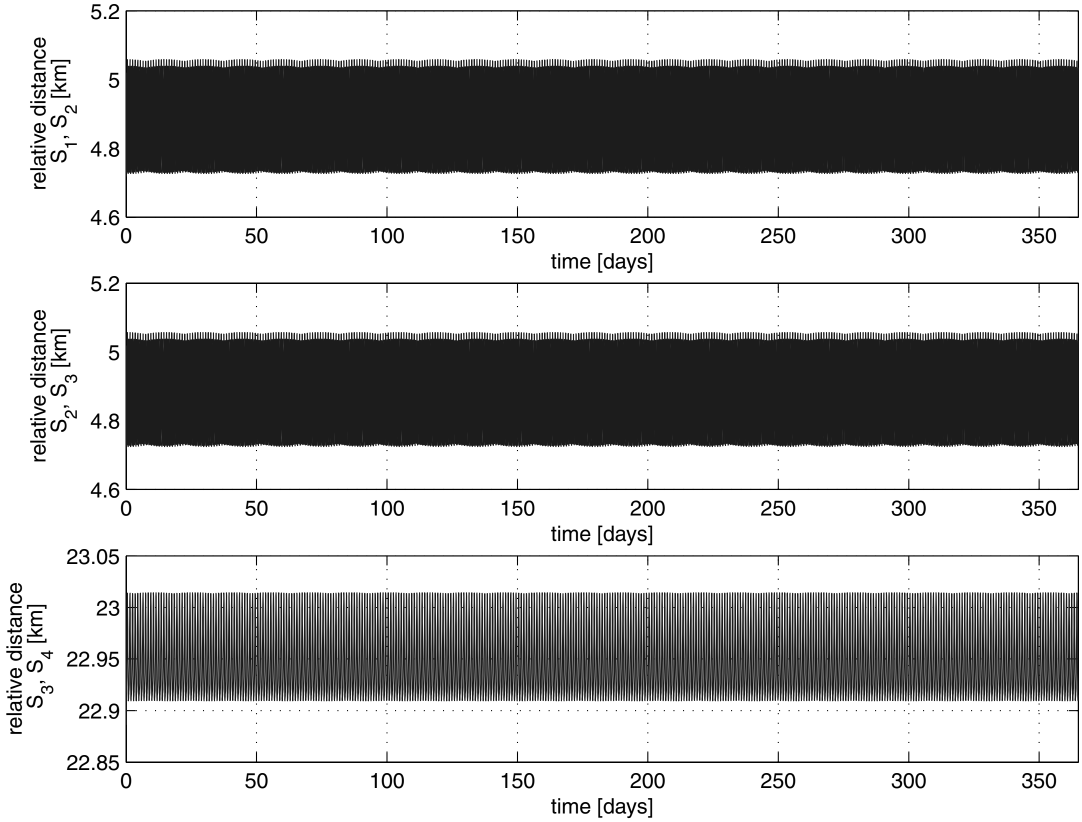

Fig. 9 Relative distance between collocated satellites subject to a J_2 perturbation. The distance is bounded during the 1-year simulation, guaranteeing collision-free operation.

elements were used for modeling the relative motion and for detecting optimal impulsive collocation maneuvers. The minimum distance between satellites that guarantees collision-free motion was taken into account to find necessary and sufficient conditions for the relative eccentricity and inclination vectors required for safe collocation.

The collocation problem was formulated using relative orbital element corrections, an approach that introduces an excess freedom into the algebraic equations modeling the relative impulsive dynamics of the satellites. This excess freedom can be used for optimizing some performance measure. However, because GEO satellites must be placed at the GEO altitude, at least the final semimajor axis must be prespecified. In the proposed framework, this was achieved by a virtual leader (i.e., a point in space that is treated as a virtual vehicle for which the orbital elements are used to steer the entire group into the GEO deadband).

Unlike previous efforts, the proposed collocation strategy treated the satellites as a formation of spacecraft and used the Gauss variational equations and synchronous orbital elements to model the relative satellite motion. The necessary corrective maneuvers were determined directly from the minimum-norm solution to the system of linear equations. The control variables are the impulsive maneuvers, and the constraints are placed on the relative orbital elements.

The effort described herein constitutes a first step in a new approach to satellite collocation. Future steps would involve a high-fidelity modeling of orbital perturbations, including tesseral resonance, lunisolar gravity, and solar radiation pressure.

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